LESSON 4-2

Degrees and Radians

Then

- You used the measures of acute angles in triangles given in degrees. (Lesson 4-1)

Now

1. Convert degree measures of angles to radian measures, and vice versa.
2. Use angle measures to solve real-world problems.

Why?

- In Lesson 4-1, you worked only with acute angles, but angles can have *any* real number measurement. For example, in skateboarding, a 540 is an aerial trick in which a skateboarder and the board rotate through an angle of 540°, or one and a half complete turns, in midair.

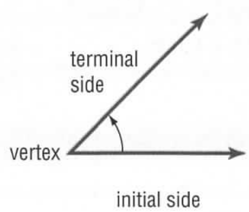
NewVocabulary

vertex
initial side
terminal side
standard position
radian
coterminal angles
linear speed
angular speed
sector

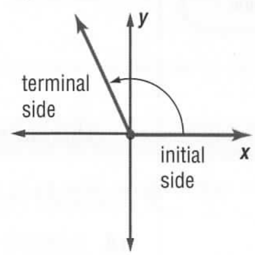
1 Angles and Their Measures

From geometry, you may recall an angle being defined as two noncollinear rays that share a common endpoint known as a **vertex**. An angle can also be thought of as being formed by the action of rotating a ray about its endpoint. From this dynamic perspective, the starting position of the ray forms the **initial side** of the angle, while the ray's position after rotation forms the angle's **terminal side**. In the coordinate plane, an angle with its vertex at the origin and its initial side along the positive x-axis is said to be in **standard position**.

Angle

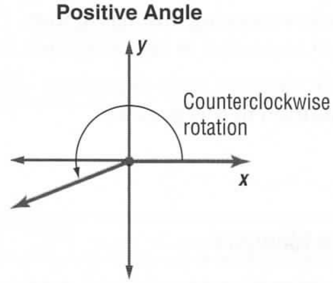


Angle in Standard Position

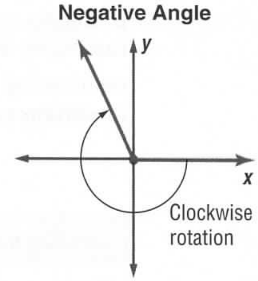


The measure of an angle describes the amount and direction of rotation necessary to move from the initial side to the terminal side of the angle. A *positive angle* is generated by a counterclockwise rotation and a *negative angle* by a clockwise rotation.

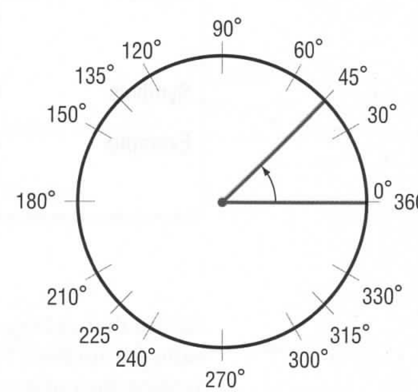
Positive Angle



Negative Angle



The most common angular unit of measure is the *degree* (°), which is equivalent to $\frac{1}{360}$ of a full rotation (counterclockwise) about the vertex. From the diagram shown, you can see that 360° corresponds to 1 complete rotation, 180° to a $\frac{1}{2}$ rotation, 90° to a $\frac{1}{4}$ rotation, and so on, as marked along the circumference of the circle.



StudyTip

Base 60 The concept of degree measurement dates back to the ancient Babylonians, who made early astronomical calculations using their number system, which was based on 60 (sexagesimal) rather than on 10 (decimal) as we do today.

Degree measures can also be expressed using a decimal degree form or a degree-minute-second (DMS) form where each degree is subdivided into 60 minutes (′) and each minute is subdivided into 60 seconds (″).

Example 1 Convert Between DMS and Decimal Degree Form

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth.

a. 56.735°

First, convert 0.735° into minutes and seconds.

$$56.735° = 56° + 0.735°\left(\frac{60'}{1°}\right) \qquad 1° = 60'$$
$$= 56° + 44.1' \qquad \text{Simplify.}$$

Next, convert 0.1′ into seconds.

$$56.735° = 56° + 44' + 0.1'\left(\frac{60''}{1'}\right) \qquad 1' = 60''$$
$$= 56° + 44' + 6'' \qquad \text{Simplify.}$$

Therefore, 56.735° can be written as 56° 44′ 6″.

TechnologyTip

DMS Form You can use some calculators to convert decimal degree values to degrees, minutes, and seconds using the DMS function under the Angle menu.

b. 32° 5′ 28″

Each minute is $\frac{1}{60}$ of a degree and each second is $\frac{1}{60}$ of a minute, so each second is $\frac{1}{3600}$ of a degree.

$$32° 5' 28'' = 32° + 5'\left(\frac{1°}{60'}\right) + 28''\left(\frac{1°}{3600''}\right) \qquad 1' = \frac{1}{60}(1°) \text{ and } 1'' = \frac{1}{3600}(1°)$$
$$\approx 32° + 0.083 + 0.008 \qquad \text{Simplify.}$$
$$\approx 32.091° \qquad \text{Add.}$$

Therefore, 32° 5′ 28″ can be written as about 32.091°.

GuidedPractice

1A. 213.875° **1B.** 89° 56′ 7″

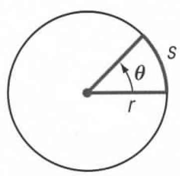
Measuring angles in degrees is appropriate when applying trigonometry to solve many real-world problems, such as those in surveying and navigation. For other applications with trigonometric functions, using an angle measured in degrees poses a significant problem. A degree has no relationship to any linear measure; inch-degrees or $\frac{\text{inch}}{\text{degree}}$ has no meaning. Measuring angles in **radians** provides a solution to this problem.

KeyConcept Radian Measure

Words The measure θ in radians of a central angle of a circle is equal to the ratio of the length of the intercepted arc s to the radius r of the circle.

Symbols $\theta = \frac{s}{r}$, where θ is measured in radians (rad)

Example A central angle has a measure of 1 radian if it intercepts an arc with the same length as the radius of the circle.



$\theta = 1$ radian when $s = r$.

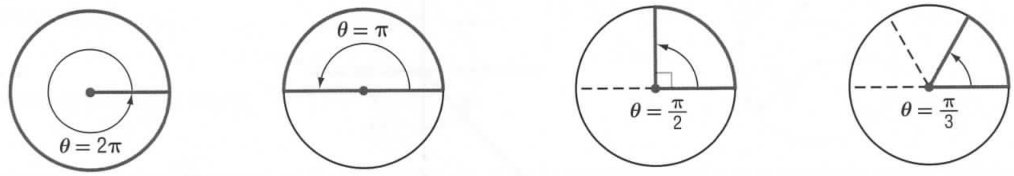
Notice that as long as the arc length s and radius r are measured using the same linear units, the ratio $\frac{s}{r}$ is unitless. For this reason, the word *radian* or its abbreviation *rad* is usually omitted when writing the radian measure of an angle.

StudyTip

Degree-Radian Equivalences From the equivalence statement $180° = \pi$ rad, you can determine $1° \approx 0.017$ rad and 1 rad $\approx 57.296°$.

The central angle representing one full rotation counterclockwise about a vertex corresponds to an arc length equivalent to the circumference of the circle, $2\pi r$. From this, you can obtain the following radian measures.

$$1 \text{ rotation} = \frac{2\pi r}{r} = 2\pi \text{ rad} \qquad \frac{1}{2} \text{ rotation} = \frac{1}{2} \cdot 2\pi = \pi \text{ rad} \qquad \frac{1}{4} \text{ rotation} = \frac{1}{4} \cdot 2\pi = \frac{\pi}{2} \text{ rad} \qquad \frac{1}{6} \text{ rotation} = \frac{1}{6} \cdot 2\pi = \frac{\pi}{3} \text{ rad}$$



ReadingMath

Angle Measure If no units of angle measure are specified, radian measure is implied. If degrees are intended, the degree symbol (°) must be used.

Because 2π radians and 360° both correspond to one complete revolution, you can write $360° = 2\pi$ radians or $180° = \pi$ radians. This last equation leads to the following equivalence statements.

$$1° = \frac{\pi}{180} \text{ radians} \qquad \text{and} \qquad 1 \text{ radian} = \left(\frac{180}{\pi}\right)^{\circ}$$

Using these statements, we obtain the following conversion rules.

KeyConcept Degree/Radian Conversion Rules

1. To convert a degree measure to radians, multiply by $\frac{\pi \text{ radians}}{180°}$.
2. To convert a radian measure to degrees, multiply by $\frac{180°}{\pi \text{ radians}}$.

Example 2 Convert Between Degree and Radian Measure

Write each degree measure in radians as a multiple of π and each radian measure in degrees.

a. 120°

$120° = 120° \left(\frac{\pi \text{ radians}}{180°}\right)$ — Multiply by $\frac{\pi \text{ radians}}{180°}$.

$= \frac{2\pi}{3}$ radians or $\frac{2\pi}{3}$ — Simplify.

b. −45°

$-45° = -45° \left(\frac{\pi \text{ radians}}{180°}\right)$ — Multiply by $\frac{\pi \text{ radians}}{180°}$.

$= -\frac{\pi}{4}$ radians or $-\frac{\pi}{4}$ — Simplify.

c. $\frac{5\pi}{6}$

$\frac{5\pi}{6} = \frac{5\pi}{6}$ radians — Multiply by $\frac{180°}{\pi \text{ radians}}$.

$= \frac{5\pi}{6} \text{ radians} \left(\frac{180°}{\pi \text{ radians}}\right)$ or 150° — Simplify.

d. $-\frac{3\pi}{2}$

$-\frac{3\pi}{2} = -\frac{3\pi}{2}$ radians — Multiply by $\frac{180°}{\pi \text{ radians}}$.

$= -\frac{3\pi}{2} \text{ radians} \left(\frac{180°}{\pi \text{ radians}}\right)$ or −270° — Simplify.

GuidedPractice

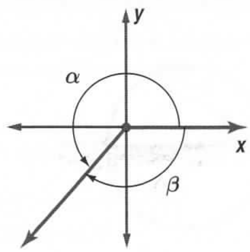
2A. 210° **2B.** −60° **2C.** $\frac{4\pi}{3}$ **2D.** $-\frac{\pi}{6}$

ReadingMath

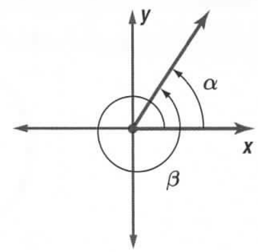
Naming Angles In trigonometry, angles are often labeled using Greek letters, such as α (alpha), β (beta), and θ (theta).

By defining angles in terms of their rotation about a vertex, two angles can have the same initial and terminal sides but different measures. Such angles are called **coterminal angles**. In the figures below, angles α and β are coterminal.

Positive and Negative Coterminal Angles



Positive Coterminal Angles



The two positive coterminal angles shown differ by one full rotation. A given angle has infinitely many coterminal angles found by adding or subtracting integer multiples of 360° or 2π radians.

KeyConcept Coterminal Angles

Degrees	Radians
If α is the degree measure of an angle, then all angles measuring $\alpha + 360n°$, where n is an integer, are coterminal with α.	If α is the radian measure of an angle, then all angles measuring $\alpha + 2n\pi$, where n is an integer, are coterminal with α.

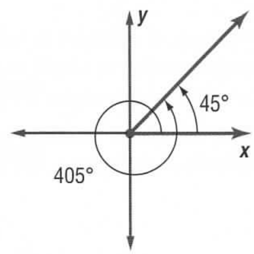
Example 3 Find and Draw Coterminal Angles

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle.

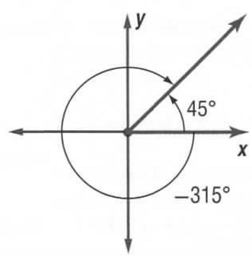
a. 45°

All angles measuring $45° + 360n°$ are coterminal with a 45° angle. Let $n = 1$ and -1.

$45° + 360(1)° = 45° + 360°$ or $405°$



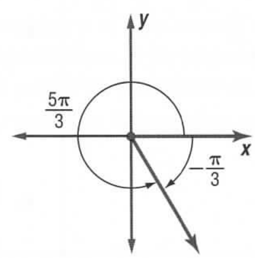
$45° + 360(-1)° = 45° - 360°$ or $-315°$



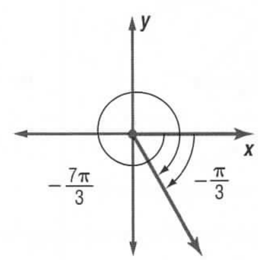
b. $-\frac{\pi}{3}$

All angles measuring $-\frac{\pi}{3} + 2n\pi$ are coterminal with a $-\frac{\pi}{3}$ angle. Let $n = 1$ and -1.

$-\frac{\pi}{3} + 2(1)\pi = -\frac{\pi}{3} + 2\pi$ or $\frac{5\pi}{3}$



$-\frac{\pi}{3} + 2(-1)\pi = -\frac{\pi}{3} - 2\pi$ or $-\frac{7\pi}{3}$



GuidedPractice

3A. $-30°$

3B. $\frac{3\pi}{4}$

2 Applications with Angle Measure

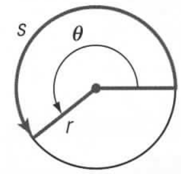
Solving $\theta = \frac{s}{r}$ for the arc length s yields a convenient formula for finding the length of an arc of a circle.

KeyConcept Arc Length

If θ is a central angle in a circle of radius r, then the length of the intercepted arc s is given by

$$s = r\theta,$$

where θ is measured in radians.



When θ is measured in degrees, you could also use the equation $s = \frac{\pi r \theta}{180}$, which already incorporates the degree-radian conversion.

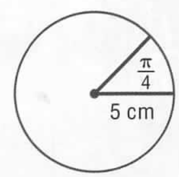
StudyTip

Operating with Radians Notice in Example 4a that when $r = 5$ centimeters and $\theta = \frac{\pi}{4}$ radians, $s = \frac{5\pi}{4}$ centimeters, not $\frac{5\pi}{4}$ centimeter-radians. This is because a radian is a unitless ratio.

Example 4 Find Arc Length

Find the length of the intercepted arc in each circle with the given central angle measure and radius. Round to the nearest tenth.

a. $\frac{\pi}{4}$, $r = 5$ cm



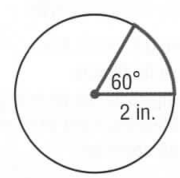
$s = r\theta$ — Arc length

$= 5\left(\frac{\pi}{4}\right)$ — $r = 5$ and $\theta = \frac{\pi}{4}$

$= \frac{5\pi}{4}$ — Simplify.

The length of the intercepted arc is $\frac{5\pi}{4}$ or about 3.9 centimeters.

b. $60°$, $r = 2$ in.



Method 1 Convert 60° to radian measure, and then use $s = r\theta$ to find the arc length.

$60° = 60°\left(\frac{\pi \text{ radians}}{180°}\right)$ — Multiply by $\frac{\pi \text{ radians}}{180°}$.

$= \frac{\pi}{3}$ — Simplify.

Substitute $r = 2$ and $\theta = \frac{\pi}{3}$.

$s = r\theta$ — Arc length

$= 2\left(\frac{\pi}{3}\right)$ — $r = 2$ and $\theta = \frac{\pi}{3}$

$= \frac{2\pi}{3}$ — Simplify.

Method 2 Use $s = \frac{\pi r \theta}{180°}$ to find the arc length.

$s = \frac{\pi r \theta}{180°}$ — Arc length

$= \frac{\pi(2)(60°)}{180°}$ — $r = 2$ and $\theta = 60°$

$= \frac{2\pi}{3}$ — Simplify.

The length of the intercepted arc is $\frac{2\pi}{3}$ or about 2.1 inches.

GuidedPractice

4A. $\frac{2\pi}{3}$, $r = 2$ m

4B. $135°$, $r = 0.5$ ft

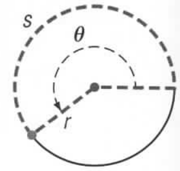
The formula for arc length can be used to analyze circular motion. The rate at which an object moves along a circular path is called its **linear speed**. The rate at which the object *rotates* about a fixed point is called its **angular speed**. Linear speed is measured in units like miles per hour, while angular speed is measured in units like revolutions per minute.

KeyConcept Linear and Angular Speed

Suppose an object moves at a constant speed along a circular path of radius r.

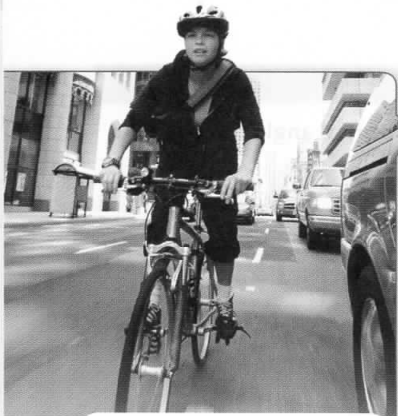
If s is the arc length traveled by the object during time t, then the object's *linear speed* v is given by $v = \frac{s}{t}$.

If θ is the angle of rotation (in radians) through which the object moves during time t, then the *angular speed* ω of the object is given by $\omega = \frac{\theta}{t}$.



ReadingMath

Omega The lowercase Greek letter omega ω is usually used to denote angular speed.



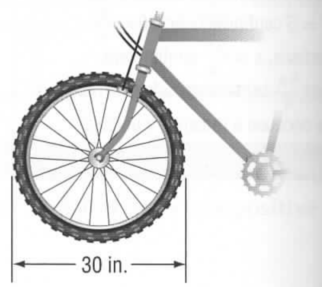
Real-WorldLink

In some U.S. cities, it is possible for bicycle messengers to ride an average of 30 to 35 miles a day while making 30 to 45 deliveries.

Source: New York Bicycle Messenger Association

Real-World Example 5 Find Angular and Linear Speeds

BICYCLING **A bicycle messenger rides the bicycle shown.**



a. During one delivery, the tires rotate at a rate of 140 revolutions per minute. Find the angular speed of the tire in radians per minute.

Because each rotation measures 2π radians, 140 revolutions correspond to an angle of rotation θ of $140 \times 2\pi$ or 280π radians.

$\omega = \frac{\theta}{t}$ — Angular speed

$= \frac{280\pi \text{ radians}}{1 \text{ minute}}$ — $\theta = 280\pi$ radians and $t = 1$ minute

Therefore, the angular speed of the tire is 280π or about 879.6 radians per minute.

b. On part of the trip to the next delivery, the tire turns at a constant rate of 2.5 revolutions per second. Find the linear speed of the tire in miles per hour.

A rotation of 2.5 revolutions corresponds to an angle of rotation θ of $2.5 \times 2\pi$ or 5π.

$v = \frac{s}{t}$ — Linear speed

$= \frac{r\theta}{t}$ — $s = r\theta$

$= \frac{15(5\pi) \text{ inches}}{1 \text{ second}}$ or $\frac{75\pi \text{ inches}}{1 \text{ second}}$ — $r = 15$ inches, $\theta = 5\pi$ radians, and $t = 1$ second

Use dimensional analysis to convert this speed from inches per second to miles per hour.

$$\frac{75\pi \text{ inches}}{1 \text{ second}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{1 \text{ foot}}{12 \text{ inches}} \times \frac{1 \text{ mile}}{5280 \text{ feet}} \approx \frac{13.4 \text{ miles}}{\text{hour}}$$

Therefore, the linear speed of the tire is about 13.4 miles per hour.

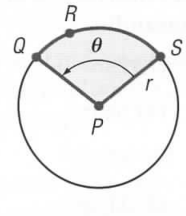
GuidedPractice

MEDIA **Consider the DVD shown.**

5A. Find the angular speed of the DVD in radians per second if the disc rotates at a rate of 3.5 revolutions per second.

5B. If the DVD player overheats and the disc begins to rotate at a slower rate of 3 revolutions per second, find the disc's linear speed in meters per minute.

Recall from geometry that a **sector** of a circle is a region bounded by a central angle and its intercepted arc. For example, the shaded portion in the figure is a sector of circle P. The ratio of the area of a sector to the area of a whole circle is equal to the ratio of the corresponding arc length to the circumference of the circle. Let A represent the area of the sector.



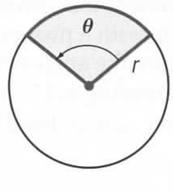
$\frac{A}{\pi r^2} = \frac{\text{length of } \widehat{QRS}}{2\pi r}$	$\frac{\text{area of sector}}{\text{area of circle}} = \frac{\text{arc length}}{\text{circumference of circle}}$
$\frac{A}{\pi r^2} = \frac{r\theta}{2\pi r}$	The length of $\widehat{QRS}$ is $r\theta$.
$A = \frac{1}{2}r^2\theta$	Solve for A.

KeyConcept Area of a Sector

The area A of a sector of a circle with radius r and central angle θ is

$$A = \frac{1}{2}r^2\theta,$$

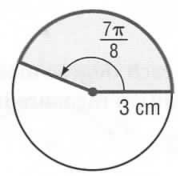
where θ is measured in radians.



Example 6 Find Areas of Sectors

a. Find the area of the sector of the circle.

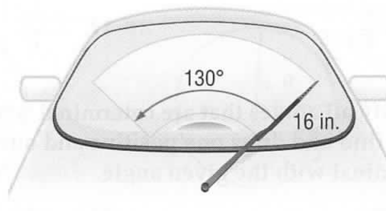
The measure of the sector's central angle θ is $\frac{7\pi}{8}$, and the radius is 3 centimeters.



$A = \frac{1}{2}r^2\theta$	Area of a sector
$= \frac{1}{2}(3)^2\left(\frac{7\pi}{8}\right)$ or $\frac{63\pi}{16}$	$r = 3$ and $\theta = \frac{7\pi}{8}$

Therefore, the area of the sector is $\frac{63\pi}{16}$ or about 12.4 square centimeters.

b. WIPERS **Find the approximate area swept by the wiper blade shown, if the total length of the windshield wiper mechanism is 26 inches.**



The area swept by the wiper blade is the difference between the areas of the sectors with radii 26 inches and 26 − 16 or 10 inches.

Convert the central angle measure to radians.

$$130° = 130°\left(\frac{\pi \text{ radians}}{180°}\right) = \frac{13\pi}{18}$$

Then use the radius of each sector to find the area swept. Let A_1 = the area of the sector with a 26-inch radius, and let A_2 = the area of the sector with a 10-inch radius.

$A = A_1 - A_2$	Swept area
$= \frac{1}{2}(26)^2\left(\frac{13\pi}{18}\right) - \frac{1}{2}(10)^2\left(\frac{13\pi}{18}\right)$	Area of a sector
$= \frac{2197\pi}{9} - \frac{325\pi}{9}$	Simplify.
$= 208\pi$ or about 653.5	Simplify.

Therefore, the swept area is about 653.5 square inches.

Real-WorldLink

A typical wipe angle for a front windshield wiper of a passenger car is about 67°. Windshield wiper blades are generally 12–30 inches long.

Source: *Car and Driver*

GuidedPractice

Find the area of the sector of a circle with the given central angle θ and radius r.

6A. $\theta = \frac{3\pi}{4}$, $r = 1.5$ ft

6B. $\theta = 50°$, $r = 6$ m

Exercises

= Step-by-Step Solutions begin on page R29.

Write each decimal degree measure in DMS form and each DMS measure in decimal degree form to the nearest thousandth. (Example 1)

1. 11.773°

2. 58.244°

3. 141.549°

4. 273.396°

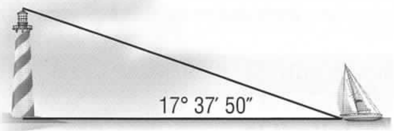
5. 87° 53′ 10″

6. 126° 6′ 34″

7. 45° 21′ 25″

8. 301° 42′ 8″

9. **NAVIGATION** A sailing enthusiast uses a sextant, an instrument that can measure the angle between two objects with a precision to the nearest 10 seconds, to measure the angle between his sailboat and a lighthouse. If his reading is 17° 37′ 50″, what is the measure in decimal degree form to the nearest hundredth? (Example 1)



Write each degree measure in radians as a multiple of π and each radian measure in degrees. (Example 2)

10. 30°

11. 225°

12. −165°

13. −45°

14. $\frac{2\pi}{3}$

15. $\frac{5\pi}{2}$

16. $-\frac{\pi}{4}$

17. $-\frac{7\pi}{6}$

Identify all angles that are coterminal with the given angle. Then find and draw one positive and one negative angle coterminal with the given angle. (Example 3)

18. 120°

19. −75°

20. 225°

21. −150°

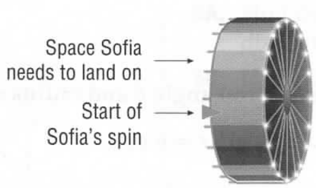
22. $\frac{\pi}{3}$

23. $-\frac{3\pi}{4}$

24. $-\frac{\pi}{12}$

25. $\frac{3\pi}{2}$

26. **GAME SHOW** Sofia is spinning a wheel on a game show. There are 20 values in equal-sized spaces around the circumference of the wheel. The value that Sofia needs to win is two spaces above the space where she starts her spin, and the wheel must make at least one full rotation for the spin to count. Describe a spin rotation in degrees that will give Sofia a winning result. (Example 3)



Find the length of the intercepted arc with the given central angle measure in a circle with the given radius. Round to the nearest tenth. (Example 4)

27. $\frac{\pi}{6}$, $r = 2.5$ m

28. $\frac{2\pi}{3}$, $r = 3$ in.

29. $\frac{5\pi}{12}$, $r = 4$ yd

30. 105°, $r = 18.2$ cm

31. 45°, $r = 5$ mi

32. 150°, $r = 79$ mm

33. **AMUSEMENT PARK** A carousel at an amusement park rotates 3024° per ride. (Example 4)

a. How far would a rider seated 13 feet from the center of the carousel travel during the ride?

b. How much farther would a second rider seated 18 feet from the center of the carousel travel during the ride than the rider in part **a**?

Find the rotation in revolutions per minute given the angular speed and the radius given the linear speed and the rate of rotation. (Example 5)

34. $\omega = \frac{2}{3}\pi \frac{\text{rad}}{\text{s}}$

35. $\omega = 135\pi \frac{\text{rad}}{\text{h}}$

36. $\omega = 104\pi \frac{\text{rad}}{\text{min}}$

37. $v = 82.3 \frac{\text{m}}{\text{s}}, 131 \frac{\text{rev}}{\text{min}}$

38. $v = 144.2 \frac{\text{ft}}{\text{min}}, 10.9 \frac{\text{rev}}{\text{min}}$

39. $v = 553 \frac{\text{in.}}{\text{h}}, 0.09 \frac{\text{rev}}{\text{min}}$

40. **MANUFACTURING** A company manufactures several circular saws with the blade diameters and motor speeds shown below. (Example 5)

Blade Diameter (in.)	Motor Speed (rps)
3	2800
5	5500
$5\frac{1}{2}$	4500
$6\frac{1}{8}$	5500
$7\frac{1}{4}$	5000

a. Determine the angular and linear speeds of the blades in each saw. Round to the nearest tenth.

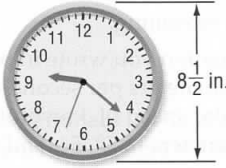
b. How much faster is the linear speed of the $6\frac{1}{8}$-inch saw compared to the 3-inch saw?

41. **CARS** On a stretch of interstate, a vehicle's tires range between 646 and 840 revolutions per minute. The diameter of each tire is 26 inches. (Example 5)

a. Find the range of values for the angular speeds of the tires in radians per minute.

b. Find the range of values for the linear speeds of the tires in miles per hour.

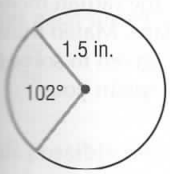
42. **TIME** A wall clock has a face diameter of $8\frac{1}{2}$ inches. The length of the hour hand is 2.4 inches, the length of the minute hand is 3.2 inches, and the length of the second hand is 3.4 inches. (Example 5)



a. Determine the angular speed in radians per hour and the linear speed in inches per hour for each hand.

b. If the linear speed of the second hand is 20 inches per minute, is the clock running fast or slow? How much time would it gain or lose per day?

GEOMETRY **Find the area of each sector.** (Example 6)

43.


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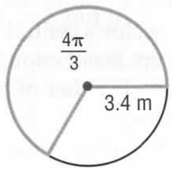

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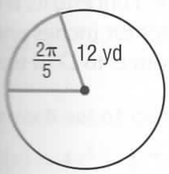

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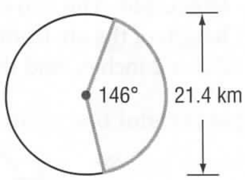
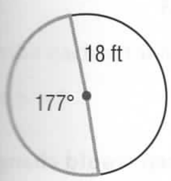
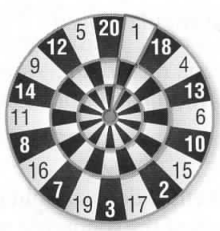


47.


48.


49. **GAMES** The dart board shown is divided into twenty equal sectors. If the diameter of the board is 18 inches, what area of the board does each sector cover? (Example 6)



50. **LAWN CARE** A sprinkler waters an area that forms one third of a circle. If the stream from the sprinkler extends 6 feet, what area of the grass does the sprinkler water? (Example 6)

The area of a sector of a circle and the measure of its central angle are given. Find the radius of the circle.

51. $A = 29$ ft^2, $\theta = 68°$

52. $A = 808$ cm^2, $\theta = 210°$

53. $A = 377$ in^2, $\theta = \frac{5\pi}{3}$

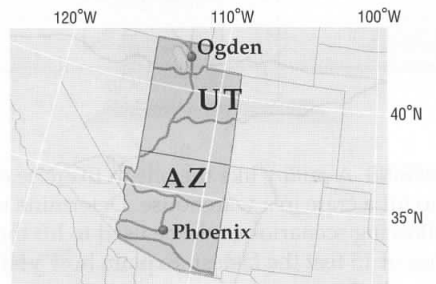
54. $A = 75$ m^2, $\theta = \frac{3\pi}{4}$

55. Describe the radian measure between 0 and 2π of an angle θ that is in standard position with a terminal side that lies in:

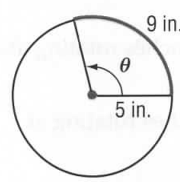
a. Quadrant I
b. Quadrant II
c. Quadrant III
d. Quadrant IV

56. If the terminal side of an angle that is in standard position lies on one of the axes, it is called a *quadrantal angle.* Give the radian measures of four quadrantal angles.

57. **GEOGRAPHY** Phoenix, Arizona, and Ogden, Utah, are located on the same line of longitude, which means that Ogden is directly north of Phoenix. The latitude of Phoenix is 33° 26′ N, and the latitude of Ogden is 41° 12′ N. If Earth's radius is approximately 3963 miles, about how far apart are the two cities?



Find the measure of angle θ in radians and degrees.

58.


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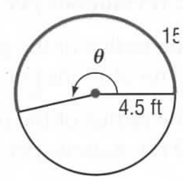

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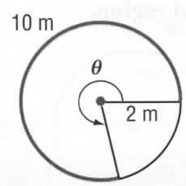
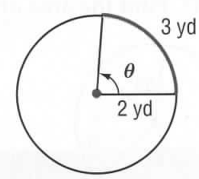
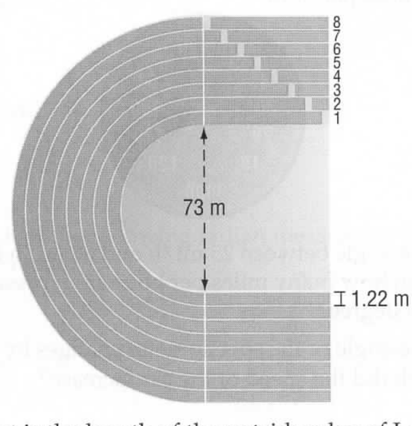


61.


62. **TRACK** The curve of a standard 8-lane track is semicircular as shown.



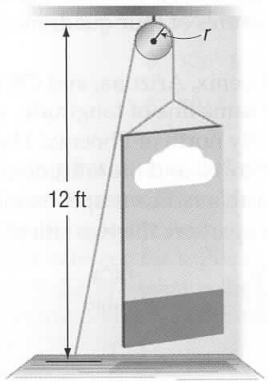
a. What is the length of the outside edge of Lane 4 in the curve?

b. How much longer is the inside edge of Lane 7 than the inside edge of Lane 3 in the curve?

63. **DRAMA** A pulley with radius r is being used to remove part of the set of a play during intermission. The height of the pulley is 12 feet.

a. If the radius of the pulley is 6 inches and it rotates 180°, how high will the object be lifted?

b. If the radius of the pulley is 4 inches and it rotates 900°, how high will the object be lifted?



64. **ENGINEERING** A pulley like the one in Exercise 63 is being used to lift a crate in a warehouse. Determine which of the following scenarios could be used to lift the crate a distance of 15 feet the fastest. Explain how you reached your conclusion.

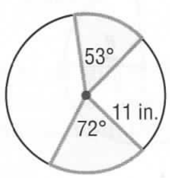
I. The radius of the pulley is 5 inches rotating at 65 revolutions per minute.

II. The radius of the pulley is 4.5 inches rotating at 70 revolutions per minute.

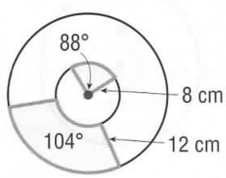
III. The radius of the pulley is 6 inches rotating at 60 revolutions per minute.

GEOMETRY **Find the area of each shaded region.**

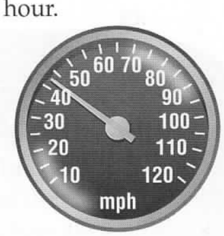
65.



66.



67. **CARS** The speedometer shown measures the speed of a car in miles per hour.



a. If the angle between 25 mi/h and 60 mi/h is 81.1°, about how many miles per hour are represented by each degree?

b. If the angle of the speedometer changes by 95°, how much did the speed of the car increase?

Find the complement and supplement of each angle, if possible. If not possible, explain your reasoning.

68. $\frac{2\pi}{5}$ **69.** $\frac{5\pi}{6}$ **70.** $\frac{3\pi}{8}$ **71.** $-\frac{\pi}{3}$

72. **SKATEBOARDING** A physics class conducted an experiment to test three different wheel sizes on a skateboard with constant angular speed.

a. Write an equation for the linear speed of the skateboard in terms of the radius and angular speed. Explain your reasoning.

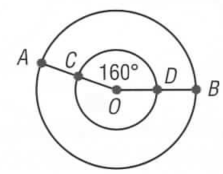
b. Using the equation you wrote in part **a**, predict the linear speed in meters per second of a skateboard with an angular speed of 3 revolutions per second for wheel diameters of 52, 56, and 60 millimeters.

c. Based on your results in part **b**, how do you think wheel size affects linear speed?

H.O.T. Problems Use Higher-Order Thinking Skills

73. **ERROR ANALYSIS** Sarah and Mateo are told that the perimeter of a sector of a circle is 10 times the length of the circle's radius. Sarah thinks that the radian measure of the sector's central angle is 8 radians. Mateo thinks that there is not enough information given to solve the problem. Is either of them correct? Explain your reasoning.

74. **CHALLENGE** The two circles shown are concentric. If the length of the arc from A to B measures 8π inches and $DB = 2$ inches, find the arc length from C to D in terms of π.



REASONING **Describe how the linear speed would change for each parameter below. Explain.**

75. a decrease in the radius

76. a decrease in the unit of time

77. an increase in the angular speed

78. **PROOF** If $\frac{s_1}{r_1} = \frac{s_2}{r_2}$, prove that $\theta_1 = \theta_2$.

79. **REASONING** What effect does doubling the radius of a circle have on each of the following measures? Explain your reasoning.

a. the perimeter of the sector of the circle with a central angle that measures θ radians

b. the area of a sector of the circle with a central angle that measures θ radians

80. **WRITING IN MATH** Compare and contrast degree and radian measures. Then create a diagram similar to the one on page 231. Label the diagram using degree measures on the inside and radian measures on the outside of the circle.

Spiral Review

Use the given trigonometric function value of the acute angle θ to find the exact values of the five remaining trigonometric function values of θ. (Lesson 4-1)

81. $\sin \theta = \frac{8}{15}$

82. $\sec \theta = \frac{4\sqrt{7}}{10}$

83. $\cot \theta = \frac{17}{19}$

84. BANKING An account that Hally's grandmother opened in 1955 earned continuously compounded interest. The table shows the balances of the account from 1955 to 1959. (Lesson 3-5)

a. Use regression to find a function that models the amount in the account. Use the number of years after Jan. 1, 1955, as the independent variable.

b. Write the equation from part **a** in terms of base e.

c. What was the interest rate on the account if no deposits or withdrawals were made during the period in question?

	Date	Balance
1	*Jan. 1, 1955*	*$2137.52*
2	*Jan. 1, 1956*	*$2251.61*
3	*Jan. 1, 1957*	*$2371.79*
4	*Jan. 1, 1958*	*$2498.39*
5	*Jan. 1, 1959*	*$2631.74*

Express each logarithm in terms of ln 2 and ln 5. (Lesson 3-2)

85. $\ln \frac{25}{16}$

86. $\ln 250$

87. $\ln \frac{10}{25}$

List all possible rational zeros of each function. Then determine which, if any, are zeros. (Lesson 2-4)

88. $f(x) = x^4 - x^3 - 12x - 144$

89. $g(x) = x^3 - 5x^2 - 4x + 20$

90. $g(x) = 6x^4 + 35x^3 - x^2 - 7x - 1$

Write each set of numbers in set-builder and interval notation if possible. (Lesson 1-3)

91. $f(x) = 4x^5 + 2x^4 - 3x - 1$

92. $g(x) = -x^6 + x^4 - 5x^2 + 4$

93. $h(x) = -\frac{1}{x^3} + 2$

Describe each set using interval notation. (Lesson 0-1)

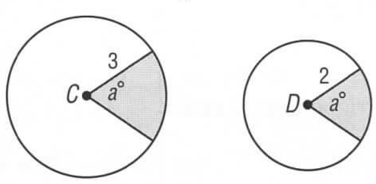
94. $n > -7$

95. $-4 \le x < 10$

96. $y < 1$ or $y \ge 11$

Skills Review for Standardized Tests

97. SAT/ACT In the figure, C and D are the centers of the two circles with radii of 3 and 2, respectively. If the larger shaded region has an area of 9, what is the area of the smaller shaded region?



Note: Figure not drawn to scale.

A 3
B 4
C 5
D 7
E 8

98. REVIEW If $\cot \theta = 1$, then $\tan \theta =$

F -1
G 0
H 1
J 3

99. REVIEW If $\sec \theta = \frac{25}{7}$, then $\sin \theta =$

A $\frac{7}{25}$

B $\frac{24}{25}$

C $\frac{24}{25}$ or $-\frac{24}{25}$

D $\frac{25}{7}$

100. Which of the following radian measures is equal to 56°?

F $\frac{\pi}{15}$

G $\frac{7\pi}{45}$

H $\frac{14\pi}{45}$

J $\frac{\pi}{3}$