LESSON 1-2 Analyzing Graphs of Functions and Relations

Then

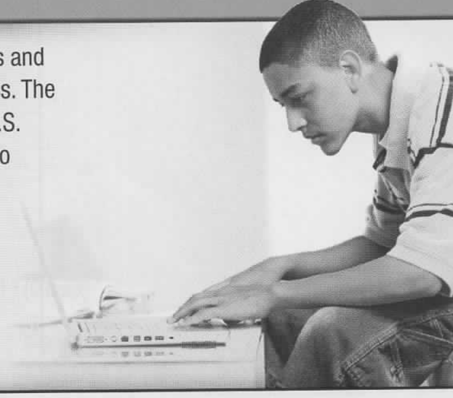
- You identified functions. (Lesson 1-1)

Now

1. Use graphs of functions to estimate function values and find domains, ranges, *y*-intercepts, and zeros of functions.
2. Explore symmetries of graphs, and identify even and odd functions.

Why?

- With more people turning to the Internet for news and entertainment, Internet advertising is big business. The total revenue *R* in millions of dollars earned by U.S. companies from Internet advertising from 1999 to 2008 can be approximated by $R(t) = 17.7t^3 - 269t^2 + 1458t - 910$, $1 \leq t \leq 10$, where *t* represents the number of years since 1998. Graphs of functions like this can help you visualize relationships between real-world quantities.

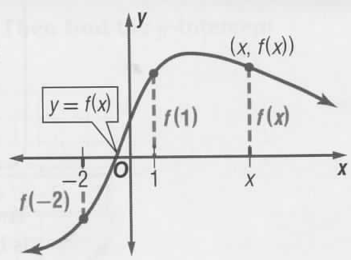


NewVocabulary

zeros
roots
line symmetry
point symmetry
even function
odd function

1 Analyzing Function Graphs

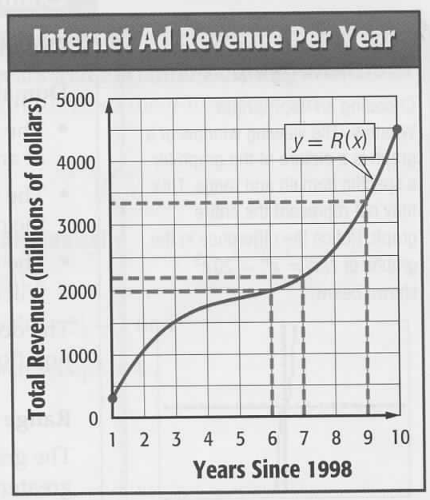
The graph of a function *f* is the set of ordered pairs $(x, f(x))$ such that *x* is in the domain of *f*. In other words, the graph of *f* is the graph of the equation $y = f(x)$. So, the value of the function is the directed distance *y* of the graph from the point *x* on the *x*-axis as shown.



You can use a graph to estimate function values.

Real-World Example 1 Estimate Function Values

INTERNET Consider the graph of function *R* shown.



a. Use the graph to estimate total Internet advertising revenue in 2007. Confirm the estimate algebraically.

The year 2007 is 9 years after 1998. The function value at $x = 9$ appears to be about \$3300 million, so the total Internet advertising revenue in 2007 was about \$3.3 billion.

To confirm this estimate algebraically, find $f(9)$.

$$f(9) = 17.7(9)^3 - 269(9)^2 + 1458(9) - 910$$
$$\approx 3326.3 \text{ million or } 3.326 \text{ billion}$$

Therefore, the graphical estimate of \$3.3 billion is reasonable.

b. Use the graph to estimate the year in which total Internet advertising revenue reached \$2 billion. Confirm the estimate algebraically.

The value of the function appears to reach \$2 billion or \$2000 million for *x*-values between 6 and 7. So, the total revenue was nearly \$2 billion in 1998 + 6 or 2004 but had exceeded \$2 billion by the end of 1998 + 7 or 2005.

To confirm algebraically, find $f(6)$ and $f(7)$.

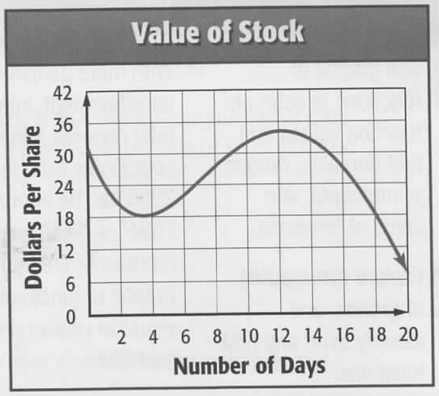
$f(6) = 17.7(6)^3 - 269(6)^2 + 1458(6) - 910$ or about 1977 million

$f(7) = 17.7(7)^3 - 269(7)^2 + 1458(7) - 910$ or about 2186 million

In billions, $f(6) \approx 1.977$ billion and $f(7) \approx 2.186$ billion. Therefore, the graphical estimate that total Internet advertising revenue reached \$2 billion in 2005 is reasonable.

GuidedPractice

1. **STOCKS** An investor assessed the average daily value of a share of a certain stock over a 20-day period. The value of the stock can be approximated by $v(d) = 0.002d^4 - 0.11d^3 + 1.77d^2 - 8.6d + 31, 0 \le d \le 20$, where d represents the day of the assessment.



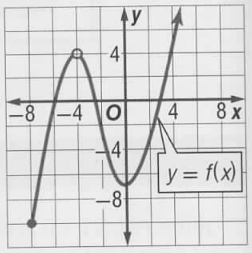
A. Use the graph to estimate the value of the stock on the 10th day. Confirm your estimate algebraically.

B. Use the graph to estimate the days during which the stock was valued at \$30 per share. Confirm your estimate algebraically.

You can also use a graph to find the domain and range of a function. Unless the graph of a function is bounded on the left by a circle or a dot, you can assume that the function extends beyond the edges of the graph.

Example 2 Find Domain and Range

Use the graph of f to find the domain and range of the function.



Domain

- The dot at $(-8, -10)$ indicates that the domain of f starts at and includes -8.
- The circle at $(-4, 4)$ indicates that -4 is not part of the domain.
- The arrow on the right side indicates that the graph will continue without bound.

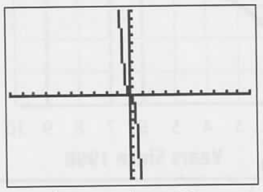
The domain of f is $[-8, -4) \cup (-4, \infty)$. In set-builder notation, the domain is $\{x \mid -8 \le x, x \ne -4, x \in \mathbb{R}\}$.

Range

The graph does not extend below $f(-8)$ or -10, but $f(x)$ increases without bound for greater and greater values of x. So, the range of f is $[-10, \infty)$.

TechnologyTip

Choosing an Appropriate Window The viewing window of a graph is a picture of the graph for a specific domain and range. This may not represent the entire graph. Notice the difference in the graphs of $f(x) = x^4 - 20x^3$ shown below.



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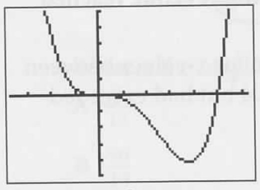

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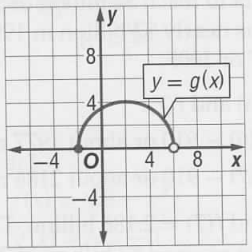
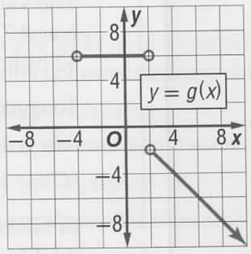
GuidedPractice

Use the graph of g to find the domain and range of each function.

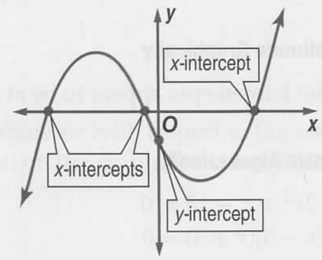
2A.



2B.



A point where a graph intersects or meets the x- or y-axis is called an intercept. An x-intercept of a graph occurs where $y = 0$. A y-intercept of a graph occurs where $x = 0$. The graph of a function can have 0, 1, or more x-intercepts, but at most one y-intercept.



To find the y-intercept of a graph of a function f algebraically, find $f(0)$.

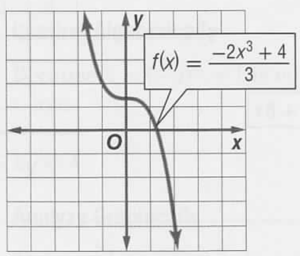
StudyTip

Labeling Axis on Graphs When you label an axis on the graph, the variable letter for the domain is on the x-axis and the variable letter for the range is on the y-axis. Throughout this book, there are many different variables used for both the domain and range. For consistency, the horizontal axis is always x and the vertical axis is always y.

Example 3 Find y-Intercepts

Use the graph of each function to approximate its y-intercept. Then find the y-intercept algebraically.

a.



Estimate Graphically

It appears that $f(x)$ intersects the y-axis at approximately $\left(0, 1\frac{1}{3}\right)$, so the y-intercept is about $1\frac{1}{3}$.

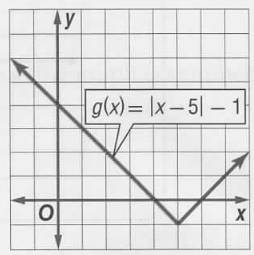
Solve Algebraically

Find $f(0)$.

$f(0) = \dfrac{-2(0)^3 + 4}{3}$ or $\dfrac{4}{3}$

The y-intercept is $\frac{4}{3}$ or $1\frac{1}{3}$.

b.



Estimate Graphically

It appears that $g(x)$ intersects the y-axis at $(0, 4)$, so the y-intercept is 4.

Solve Algebraically

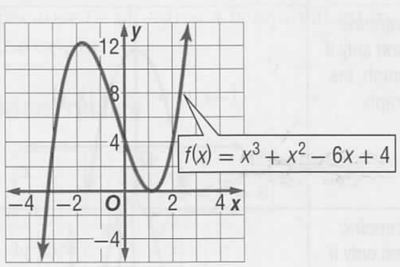
Find $g(0)$.

$g(0) = |0 - 5| - 1$ or 4

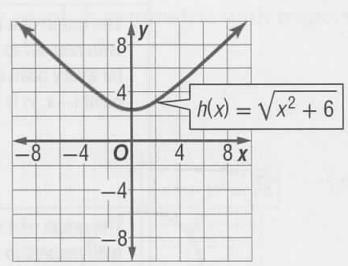
The y-intercept is 4.

GuidedPractice

3A.

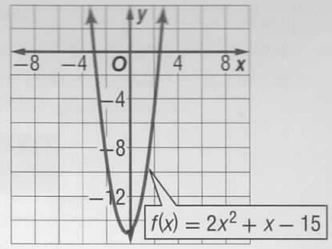


3B.



The x-intercepts of the graph of a function are also called the **zeros** of a function. The solutions of the corresponding equation are called the **roots** of the equation. To find the zeros of a function f, set the function equal to 0 and solve for the independent variable.

Use the graph of $f(x) = 2x^2 + x - 15$ to approximate its zero(s). Then find its zero(s) algebraically.



Estimate Graphically

The x-intercepts appear to be at about −3 and 2.5.

Solve Algebraically

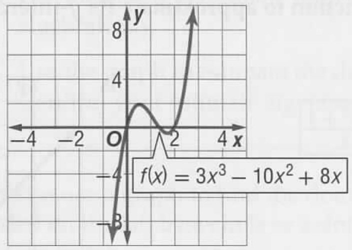
$2x^2 + x - 15 = 0$	Let $f(x) = 0$.
$(2x - 5)(x + 3) = 0$	Factor.
$2x - 5 = 0$ or $x + 3 = 0$	Zero Product Property
$x = 2.5$ $\quad x = -3$	Solve for x.

The zeros of f are −3 and 2.5.

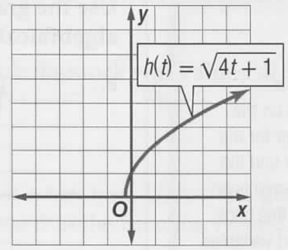
Guided Practice

Use the graph of each function to approximate its zero(s). Then find its zero(s) algebraically.

4A.



4B.



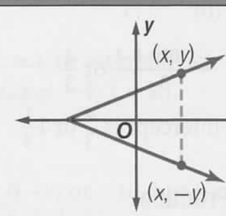
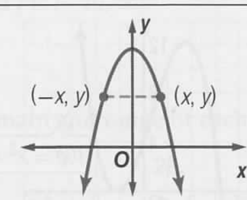
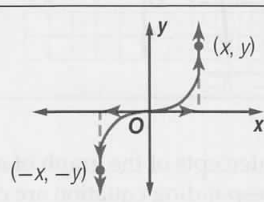
2 Symmetry of Graphs

Graphs of relations can have two different types of symmetry. Graphs with **line symmetry** can be folded along a line so that the two halves match exactly. Graphs that have **point symmetry** can be rotated 180° with respect to a point and appear unchanged. The three most common types of symmetry are shown below.

Study Tip

Symmetry, Relations, and Functions There are numerous *relations* that have x-axis, y-axis, and origin symmetry. However, the only *function* that has all three types of symmetry is the zero function, $f(x) = 0$.

Key Concept Tests for Symmetry

Graphical Test	Model	Algebraic Test
The graph of a relation is *symmetric with respect to the x-axis* if and only if for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph.		Replacing y with $-y$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the y-axis* if and only if for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph.		Replacing x with $-x$ produces an equivalent equation.
The graph of a relation is *symmetric with respect to the origin* if and only if for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph.		Replacing x with $-x$ and y with $-y$ produces an equivalent equation.

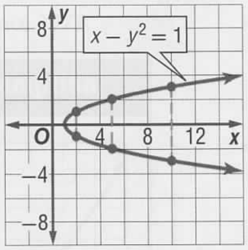
StudyTip

Symmetry It is possible for a graph to exhibit more than one type of symmetry.

Example 5 Test for Symmetry

Use the graph of each equation to test for symmetry with respect to the x-axis, y-axis, and the origin. Support the answer numerically. Then confirm algebraically.

a. $x - y^2 = 1$



Analyze Graphically

The graph appears to be symmetric with respect to the x-axis because for every point (x, y) on the graph, there is a point $(x, -y)$.

Support Numerically

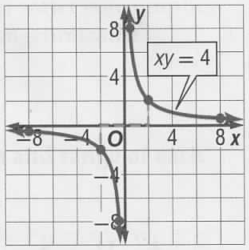
A table of values supports this conjecture.

x	2	2	5	5	10	10
y	1	−1	2	−2	3	−3
(x, y)	(2, 1)	(2, −1)	(5, 2)	(5, −2)	(10, 3)	(10, −3)

Confirm Algebraically

Because $x - (-y)^2 = 1$ is equivalent to $x - y^2 = 1$, the graph is symmetric with respect to the x-axis.

b. $xy = 4$



Analyze Graphically

The graph appears to be symmetric with respect to the origin because for every point (x, y) on the graph, there is a point $(-x, -y)$.

Support Numerically

A table of values supports this conjecture.

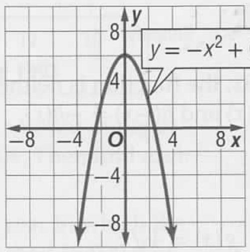
x	−8	−2	−0.5	0.5	2	8
y	−0.5	−2	−8	8	2	0.5
(x, y)	(−8, −0.5)	(−2, −2)	(−0.5, −8)	(0.5, 8)	(2, 2)	(8, 0.5)

Confirm Algebraically

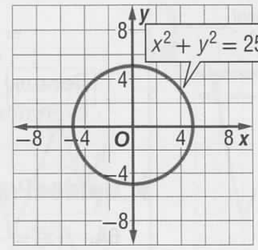
Because $(-x)(-y) = 4$ is equivalent to $xy = 4$, the graph is symmetric with respect to the origin.

GuidedPractice

5A.



5B.



Graphs of functions can have y-axis or origin symmetry. Functions with these types of symmetry have special names.

KeyConcept Even and Odd Functions

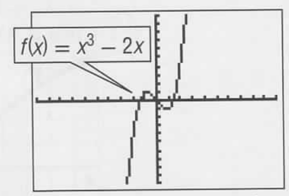
Type of Function	Algebraic Test
Functions that are symmetric with respect to the y-axis are called **even functions**.	For every x in the domain of f, $f(-x) = f(x)$.
Functions that are symmetric with respect to the origin are called **odd functions**.	For every x in the domain of f, $f(-x) = -f(x)$.

Example 6 Identify Even and Odd Functions

GRAPHING CALCULATOR Graph each function. Analyze the graph to determine whether each function is *even, odd,* or *neither.* Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.

a. $f(x) = x^3 - 2x$

It appears that the graph of the function is symmetric with respect to the origin. Test this conjecture.



[−10, 10] scl: 1 by [−10, 10] scl: 1

$f(-x) = (-x)^3 - 2(-x)$ — Substitute $-x$ for x.

$= -x^3 + 2x$ — Simplify.

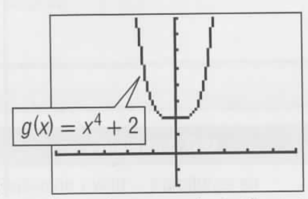
$= -(x^3 - 2x)$ — Distributive Property

$= -f(x)$ — Original function $f(x) = x^3 - 2x$

The function is odd because $f(-x) = -f(x)$. Therefore, the function is symmetric with respect to the origin.

b. $g(x) = x^4 + 2$

It appears that the graph of the function is symmetric with respect to the y-axis. Test this conjecture.



[−5, 5] scl: 1 by [−2, 8] scl: 1

$g(-x) = (-x)^4 + 2$ — Substitute $-x$ for x.

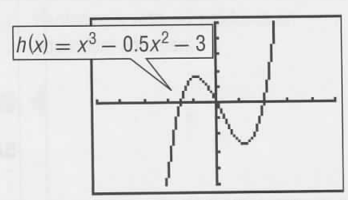
$= x^4 + 2$ — Simplify.

$= g(x)$ — Original function $g(x) = x^4 + 2$

The function is even because $g(-x) = g(x)$. Therefore, the function is symmetric with respect to the y-axis.

c. $h(x) = x^3 - 0.5x^2 - 3x$

It appears that the graph of the function may be symmetric with respect to the origin. Test this conjecture algebraically.



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$h(-x) = (-x)^3 - 0.5(-x)^2 - 3(-x)$ — Substitute $-x$ for x.

$= -x^3 - 0.5x^2 + 3x$ — Simplify.

Because $-h(x) = -x^3 + 0.5x^2 + 3x$, the function is neither even nor odd because $h(-x) \neq h(x)$ and $h(-x) \neq -h(x)$.

StudyTip

Even and Odd Functions It is important to always confirm symmetry algebraically. Graphs that appear to be symmetrical may not actually be.

GuidedPractice

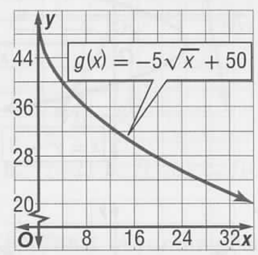
6A. $f(x) = \frac{2}{x^2}$ **6B.** $g(x) = 4\sqrt{x}$ **6C.** $h(x) = x^5 - 2x^3 + x$

Exercises

◯ = Step-by-Step Solutions begin on page R29.

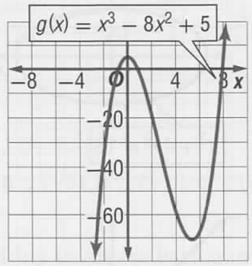
Use the graph of each function to estimate the indicated function values. Then confirm the estimate algebraically. Round to the nearest hundredth, if necessary. (Example 1)

1.



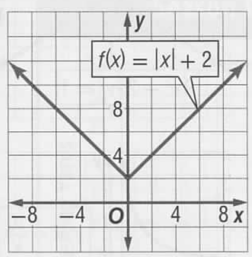
a. $g(6)$ b. $g(12)$ c. $g(19)$

2.



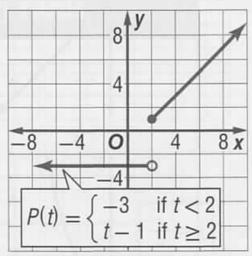
a. $g(-2)$ b. $g(1)$ c. $g(8)$

3.



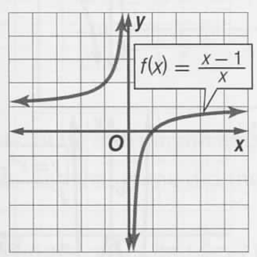
a. $f(-8)$ b. $f(-3)$ c. $f(0)$

4.



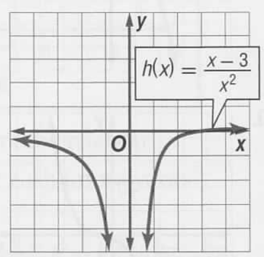
a. $P(-6)$ b. $P(2)$ c. $P(9)$

5.



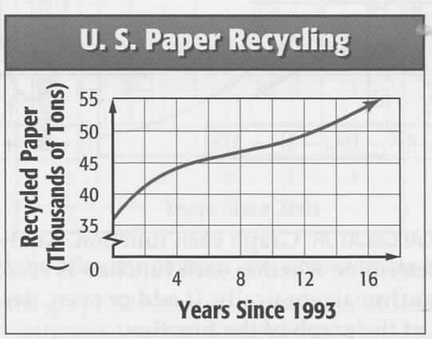
a. $f(-3)$ b. $f(0.5)$ c. $f(0)$

6.



a. $h(-1)$ b. $h(1.5)$ c. $h(2)$

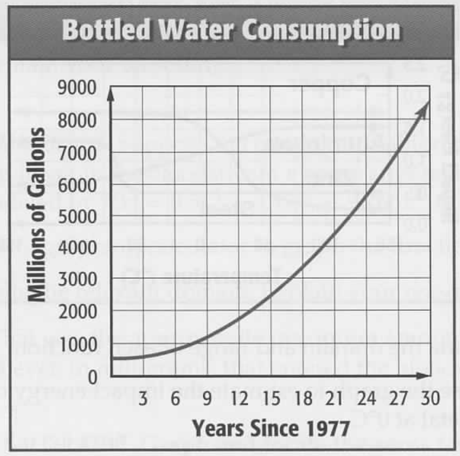
7. **RECYLING** The quantity of paper recycled in the United States in thousands of tons from 1993 to 2007 can be modeled by $p(x) = -0.0013x^4 + 0.0513x^3 - 0.662x^2 + 4.128x + 35.75$, where x is the number of years since 1993. (Example 1)



a. Use the graph to estimate the amount of paper recycled in 1993, 1999, and 2006. Then find each value algebraically.

b. Use the graph to estimate the year in which the quantity of paper recycled reached 50,000 tons. Confirm algebraically.

8. **WATER** Bottled water consumption from 1977 to 2006 can be modeled using $f(x) = 9.35x^2 - 12.7x + 541.7$, where x represents the number of years since 1977. (Example 1)



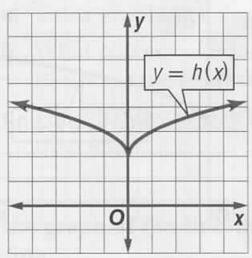
a. Use the graph to estimate the amount of bottled water consumed in 1994.

b. Find the 1994 consumption algebraically. Round to the nearest ten million gallons.

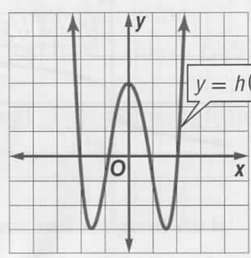
c. Use the graph to estimate when water consumption was 6 billion gallons. Confirm algebraically.

Use the graph of h to find the domain and range of each function. (Example 2)

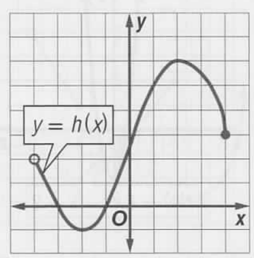
9.



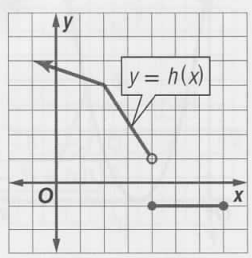
10.



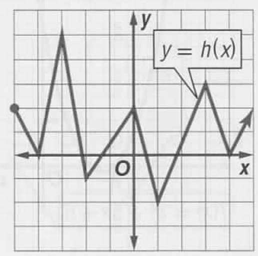
11.



12.



13.



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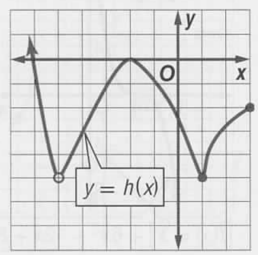
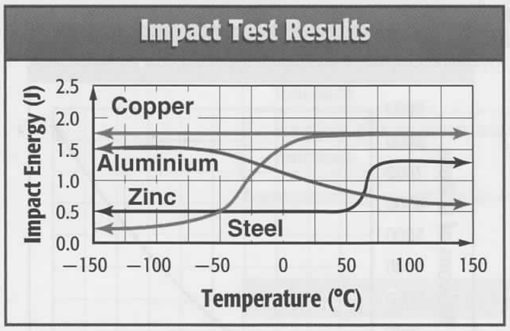


15. **ENGINEERING** Tests on the physical behavior of four metal specimens are performed at various temperatures in degrees Celsius. The impact energy, or energy absorbed by the sample during the test, is measured in joules. The test results are shown. (Example 2)

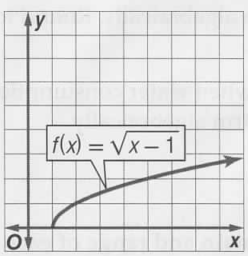


a. State the domain and range of each function.

b. Use the graph to estimate the impact energy of each metal at 0°C.

Use the graph of each function to find its y-intercept and zero(s). Then find these values algebraically. (Examples 3 and 4)

16.



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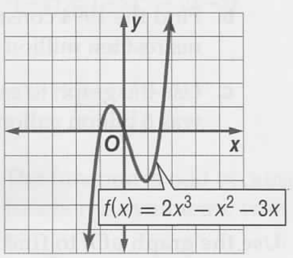

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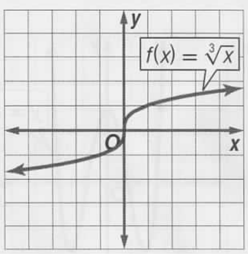

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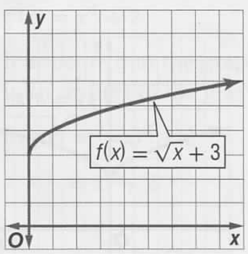

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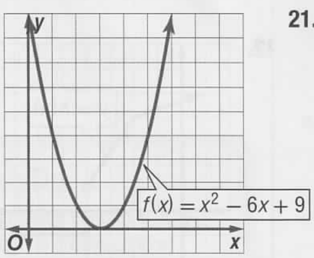

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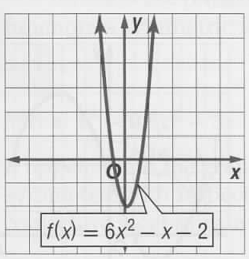

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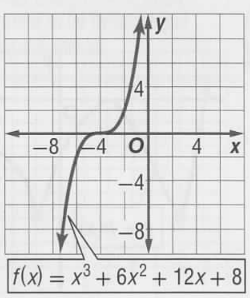

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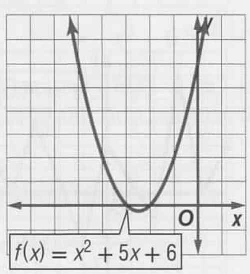
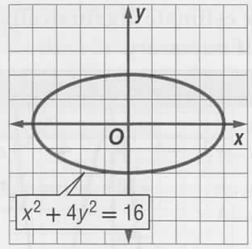


Use the graph of each equation to test for symmetry with respect to the x-axis, y-axis, and the origin. Support the answer numerically. Then confirm algebraically. (Example 5)

24.



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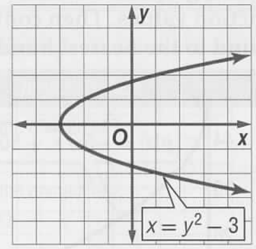

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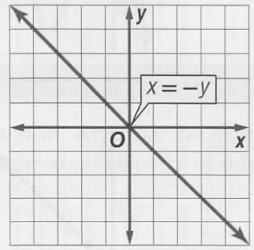

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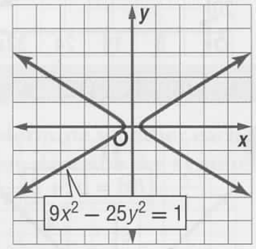

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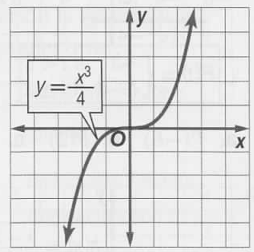

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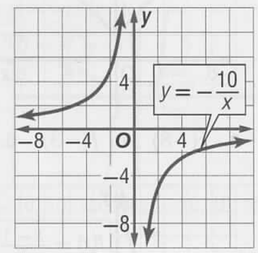

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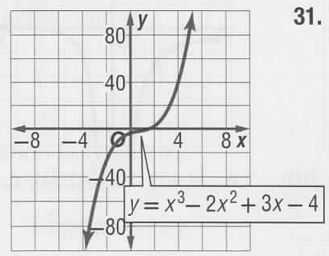

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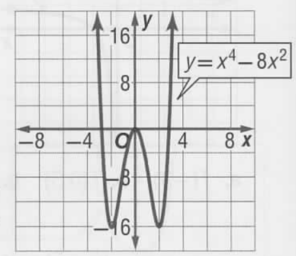

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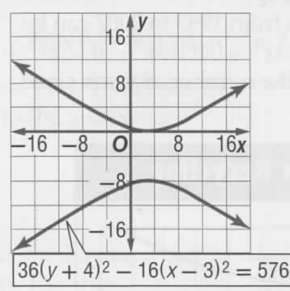

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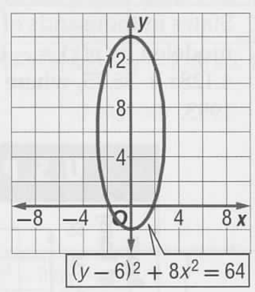


GRAPHING CALCULATOR **Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.** (Example 6)

34. $f(x) = x^2 + 6x + 10$

35. $f(x) = -2x^3 + 5x - 4$

36. $g(x) = \sqrt{x+6}$

37. $h(x) = \sqrt{x^2 - 9}$

38. $h(x) = |8 - 2x|$

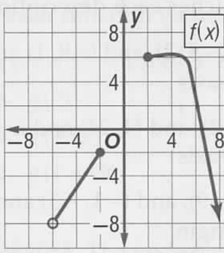
39. $f(x) = |x^3|$

40. $f(x) = \dfrac{x+4}{x-2}$

41. $g(x) = \dfrac{x^2}{x+1}$

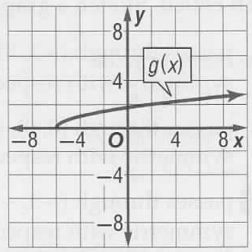
Use the graph of each function to estimate the indicated function values.

42.



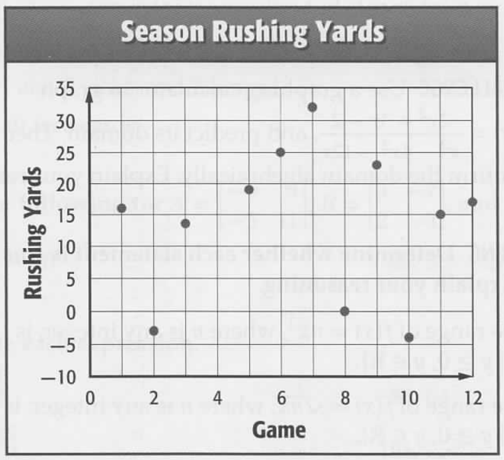
a. $f(-2)$ b. $f(-6)$ c. $f(0)$

43.



a. $g(-8)$ b. $g(-6)$ c. $g(-2)$

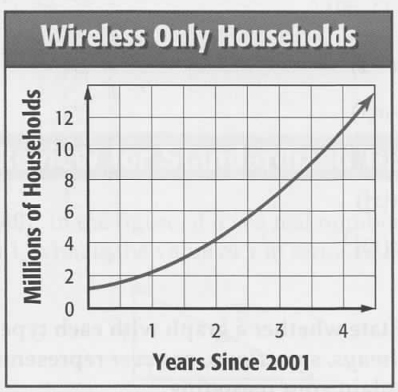
44. FOOTBALL A running back's rushing yards for each game in a season are shown.



a. State the domain and range of the relation.

b. In what game did the player rush for no yards?

45. PHONES The number of households h in millions with only wireless phone service from 2001 to 2005 can be modeled by $h(x) = 0.5x^2 + 0.5x + 1.2$, where x represents the number of years after 2001.



a. State the relevant domain and approximate the range.

b. Use the graph to estimate the number of households with only wireless phone service in 2003. Then find it algebraically.

c. Use the graph to approximate the y-intercept of the function. Then find it algebraically. What does the y-intercept represent?

d. Does this function have any zeros? If so, estimate them and explain their meaning. If not, explain why.

46. FUNCTIONS Consider $f(x) = x^n$.

a. Use a graphing calculator to graph $f(x)$ for values of n in the range $1 \le n \le 6$, where $n \in \mathbb{N}$.

b. Describe the domain and range of each function.

c. Describe the symmetry of each function.

d. Predict the domain, range, and symmetry for $f(x) = x^{35}$. Explain your reasoning.

47. PHARMACOLOGY Suppose the number of milligrams of a pain reliever in the bloodstream x hours after taking a dose is modeled by $f(x) = 0.5x^4 + 3.45x^3 - 96.65x^2 + 347.7x$.

a. Use a graphing calculator to graph the function.

b. State the relevant domain. Explain your reasoning.

c. What was the approximate maximum amount of pain reliever, in milligrams, that entered the bloodstream?

GRAPHING CALCULATOR Graph and locate the zeros for each function. Confirm your answers algebraically.

48. $f(x) = \dfrac{4x - 1}{x}$ **49.** $f(x) = \dfrac{x^2 + 9}{x + 3}$

50. $h(x) = \sqrt{x^2 + 4x + 3}$ **51.** $h(x) = 2\sqrt{x + 12} - 8$

52. $g(x) = -12 + \frac{4}{x}$ **53.** $g(x) = \frac{6}{x} + 3$

54. TELEVISION The percent of households h with basic cable for the years 1980 through 2006 can be modeled using $h(x) = -0.115x^2 + 4.43x + 25.6$, $1980 \le x \le 2006$, where x represents the number of years after 1980.

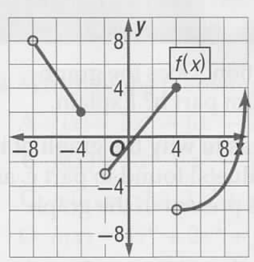
a. Use a graphing calculator to graph the function.

b. What percent of households had basic cable in 1999? Round to the nearest percent.

c. For what years was the percent of subscribers greater than 65%?

Use the graph of f to find the domain and range of each function.

55.



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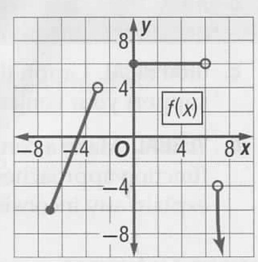

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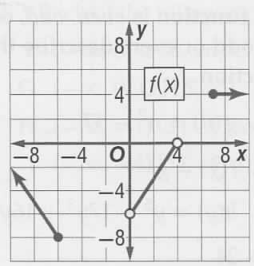

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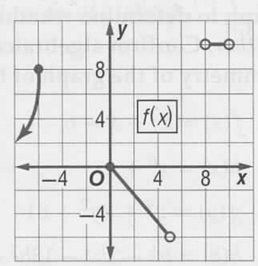
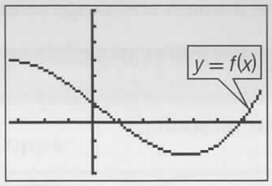


1930 to 2000 can be modeled by $f(x) = 0.0001x^3 - 0.001x^2 - 0.825x + 12.58$, where x is the number of years since 1930.



[−50, 100] scl: 15 by [−30, 70] scl: 10

a. State the relevant domain and estimate the range for this domain.

b. Use the graph to approximate the y-intercept. Then find the y-intercept algebraically. What does the y-intercept represent?

c. Find and interpret the zeros of the function.

d. Use the model to determine what the percent population change will be in 2080. Does this value seem realistic? Explain your reasoning.

60. STOCK MARKET The percent p a stock price has fluctuated in one year can be modeled by $p(x) = 0.0005x^4 - 0.0193x^3 + 0.243x^2 - 1.014x + 1.04$, where x is the number of months since January.

a. Use a graphing calculator to graph the function.

b. State the relevant domain and estimate the range.

c. Use the graph to approximate the y-intercept. Then find the y-intercept algebraically. What does the y-intercept represent?

d. Find and interpret any zeros of the function.

61. MULTIPLE REPRESENTATIONS In this problem, you will investigate the range values of $f(x) = \frac{1}{x - 2}$ as x approaches 2.

a. TABULAR Copy and complete the table below. Add an additional value to the left and right of 2.

x	1.99	1.999	2	2.001	2.01
f(x)					

b. ANALYTICAL Use the table from part **a** to describe the behavior of the function as x approaches 2.

c. GRAPHICAL Graph the function. Does the graph support your conjecture from part **b**? Explain.

d. VERBAL Make a conjecture as to why the graph of the function approaches the value(s) found in part **c**, and explain any inconsistencies present in the graph.

GRAPHING CALCULATOR Graph each function. Analyze the graph to determine whether each function is *even*, *odd*, or *neither*. Confirm algebraically. If odd or even, describe the symmetry of the graph of the function.

62. $f(x) = x^2 - x - 6$

63. $g(n) = n^2 - 37$

64. $h(x) = x^6 + 4$

65. $f(g) = g^9$

66. $g(y) = y^4 + 8y^2 + 81$

67. $h(y) = y^5 - 17y^3 + 16y$

68. $h(b) = b^4 - 2b^3 - 13b^2 + 14b + 24$

OPEN ENDED Sketch a graph that matches each description.

69. passes through (−3, 8), (−4, 4), (−5, 2), and (−8, 1) and is symmetric with respect to the y-axis

70. passes through (0, 0), (2, 6), (3, 12), and (4, 24) and is symmetric with respect to the x-axis

71. passes through (−3, −18), (−2, −9), and (−1, −3) and is symmetric with respect to the origin

72. passes through (4, −16), (6, −12), and (8, −8) and represents an even function

73. WRITING IN MATH Explain why a function can have 0, 1, or more x-intercepts but only one y-intercept.

74. CHALLENGE Use a graphing calculator to graph $f(x) = \frac{2x^2 + 3x - 2}{x^3 - 4x^2 - 12x}$, and predict its domain. Then confirm the domain algebraically. Explain your reasoning.

REASONING Determine whether each statement is *true* or *false*. Explain your reasoning.

75. The range of $f(x) = nx^2$, where n is any integer, is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

76. The range of $f(x) = \sqrt{nx}$, where n is any integer, is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

77. All odd functions are also symmetric with respect to the line $y = -x$.

78. An even function rotated $180n°$ about the origin, where n is any integer, remains an even function.

REASONING If $a(x)$ is an odd function, determine whether $b(x)$ is *odd*, *even*, *neither*, or *cannot be determined*. Explain your reasoning.

79. $b(x) = a(-x)$

80. $b(x) = -a(x)$

81. $b(x) = [a(x)]^2$

82. $b(x) = a(|x|)$

83. $b(x) = [a(x)]^3$

REASONING State whether a graph with each type of symmetry *always*, *sometimes*, or *never* represents a function. Explain your reasoning.

84. symmetric with respect to the line $x = 4$

85. symmetric with respect to the line $y = 2$

86. symmetric with respect to the line $y = x$

87. symmetric with respect to both the x- and y-axes

88. WRITING IN MATH Can a function be both even and odd? Explain your reasoning.

Spiral Review

Find each function value. (Lesson 1-1)

89. $g(x) = x^2 - 10x + 3$
- **a.** $g(2)$
- **b.** $g(-4x)$
- **c.** $g(1 + 3n)$

90. $h(x) = 2x^2 + 4x - 7$
- **a.** $h(-9)$
- **b.** $h(3x)$
- **c.** $h(2 + m)$

91. $p(x) = \frac{2x^3 + 2}{x^2 - 2}$
- **a.** $p(3)$
- **b.** $p(x^2)$
- **c.** $p(x + 1)$

92. GRADES The midterm grades for a Chemistry class of 25 students are shown. Find the measures of spread for the data set. (Lesson 0-8)

Midterm Grades				
89	76	91	72	81
81	65	74	80	74
73	92	76	83	96
66	61	80	74	70
97	78	73	62	72

93. PLAYING CARDS From a standard 52-card deck, find how many 5-card hands are possible that fit each description. (Lesson 0-7)
- **a.** 3 hearts and 2 clubs
- **b.** 1 ace, 2 jacks, and 2 kings
- **c.** all face cards

Find the following for $A = \begin{bmatrix} -6 & 3 \\ -5 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -7 \\ 2 & -3 \end{bmatrix}$, **and** $C = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$. (Lesson 0-6)

94. $4A - 2B$

95. $3C + 2A$

96. $-2(B - 3A)$

Evaluate each expression. (Lesson 0-4)

97. $27^{\frac{1}{3}}$

98. $64^{\frac{5}{6}}$

99. $49^{-\frac{1}{2}}$

100. $16^{-\frac{3}{4}}$

101. $25^{\frac{3}{2}}$

102. $36^{-\frac{3}{2}}$

103. GENETICS Suppose R and W represent two genes that a plant can inherit from its parents. The terms of the expansion of $(R + W)^2$ represent the possible pairings of the genes in the offspring. Write $(R + W)^2$ as a polynomial. (Lesson 0-3)

Simplify. (Lesson 0-2)

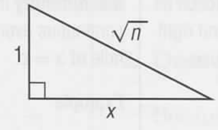
104. $(2 + i)(4 + 3i)$

105. $(1 + 4i)^2$

106. $(2 - i)(3 + 2i)(1 - 4i)$

Skills Review for Standardized Tests

107. SAT/ACT In the figure, if n is a real number greater than 1, what is the value of x in terms of n?



- A $\sqrt{n^2 - 1}$
- B $\sqrt{n - 1}$
- C $\sqrt{n + 1}$
- D $n - 1$
- E $n + 1$

108. REVIEW Which inequality describes the range of $f(x) = x^2 + 1$ over the domain $-2 < x < 3$?

- F $5 \le y < 9$
- G $2 < y < 10$
- H $1 < y < 9$
- J $1 \le y < 10$

109. Which of the following is an even function?

- A $f(x) = 2x^4 + 6x^3 - 5x^2 - 8$
- B $g(x) = 3x^6 + x^4 - 5x^2 + 15$
- C $m(x) = x^4 + 3x^3 + x^2 + 35x$
- D $h(x) = 4x^6 + 2x^4 + 6x - 4$

110. Which of the following is the domain of $g(x) = \frac{1 + x}{x^2 - 16x}$?

- F $(-\infty, 0) \cup (0, 16) \cup (16, \infty)$
- G $(-\infty, 0] \cup [16, \infty)$
- H $(-\infty, -1) \cup (-1, \infty)$
- J $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$